GEOMETRY.

492. Proposed by FRANK V. MORLEY, Student, Haverford College.

Let a_i $(i = 1, 2, 3, 4)$ be four points on a circle, and let the symmedian point of the triangle formed by omitting a_i be s_i. Prove that the four points s_i have the same diagonal triangle as the four points a_i.

493. Proposed by FLORENCE P. LEWIS, Goucher College, Baltimore.

Construct three circles each of which shall be tangent to the other two and to two sides of a given triangle.

494. Proposed by DAVID F. BARROW, University of Texas.

Students of geometry are very apt to assume that a theorem, true in general, will hold in all limiting cases. This trustfulness leads to frequent errors. An example is the following: Let C_1, C_2, C_3, C_4 denote four circles, and P_{ij}, P_{ij}' denote the two points in which C_i and C_j intersect. If P_{12}, P_{23}, P_{34}, P_{41} are concyclic on a circle C, then P_{12}', P_{23}', P_{34}', P_{41}' will be concyclic on a circle C'. This is still true if C is very small. Hence we might hastily conclude that: If four circles are concurrent, then their other intersections, taken in pairs in a cyclic order, are concyclic. Why is not this true?

CALCULUS.

410. Proposed by J. A. BULLARD, Worcester, Massachusetts.

(*a*) Find the area of the loop of the curve $x^{2q+1} + y^{2q+1} = (2q + 1)ax^qy^q$. (For $q = 1$, we have the folium of Calculus Problem No. 379.)

(*b*) Find the area between the curve and its asymptote.

[From Johnson's *Integral Calculus*.]

411. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Prove that the volume bounded by the surface $f(x, y, z) = 0$ is $\frac{1}{3}\int\int\left(z - x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}\right)dxdy$ integrated over the area determined by projecting the surface on the xy-plane.

412. Proposed by CLIFFORD N. MILLS, Brookings, S. Dakota.

Given a triangular field of sides a, b, and c. Show how to divide the field into two equal parts by a straight fence so that the cost of the fence is the least.

MECHANICS.

328. Proposed by JOSEPH B. REYNOLDS, Lehigh University.

Find the envelope of all possible trajectories when a particle is projected with a constant velocity v from a fixed point at a distance a from the center of attraction under the law of gravitation.

329. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

A smooth circular table is surrounded by a smooth vertical rim. Show that the ball, whose coefficient of restitution is e, projected along the table from a point in the rim in a direction making an angle $\tan^{-1} e^{\frac{3}{2}}$ with the radius through the point, will return to the point of projection after three rebounds.

NUMBER THEORY.

246. Proposed by ALBERT A. BENNETT, Princeton University.

Prove that

$$\frac{1}{\sqrt{b}}\left[\left(\frac{a+\sqrt{b}}{2}\right)^n - \left(\frac{a-\sqrt{b}}{2}\right)^n\right]$$

is an integer for every positive integral value of n, whenever a is an odd integer, positive or negative, and $b \equiv 1 \pmod 4$.